

Homework 5: AE601, Computational Fluid Mechanics

November. 8 2006

Due: November 15, 2006

Problem 1.

We consider the solution of

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 4\pi \sin(2\pi x) \left[\pi \cos(2\pi y^2)(1 + 4y^2) + \sin(2\pi y^2) \right]$$

on the unit square $[0,1] \times [0,1]$. We specify Dirichlet boundary conditions according to the following analytical solution

$$u(x, y) = \sin(2\pi x) \cos(2\pi y^2)$$

and solve with the 5-point central difference scheme (CDS in x-direction and CDS in y-direction).

We consider an equidistant grid with 16 grid points in x and 16 grid points in y -direction. The resulting system of algebraic equations has the form

$$A * \vec{u} = \vec{b}$$

where A is 196x196 matrix. Why is the size of the matrix 196x196?

- Discretize the Poisson equation and cast into the matrix-vector notation.
- Solve the system with a direct method ($u = A \backslash b$ in Matlab). Compare the solution to the analytical solution and determine the discretization error.
- Solve the system with the Gauss-Seidel method. Determine the iteration error by comparing the iterative solution to the solution determined with the direct method.