AE 601 - Computational Fluid Mechanics **MIDTERM PROJECT** Time Developing Temperature in a Parallel Plate Couette Flow

Project report due on March 27, 2007

Consider the time evolution of the temperature field initially at a constant temperature in a laminar flow between two parallel plates separated by a distance 2h. Consider air as the medium. The bottom plate at y=-h is at rest. The top plate at y=h moves at constant velocity of U. The analytical fluid velocity profile is given by $u(y) = \frac{U}{2}(1+\frac{y}{h})$ The energy equation for this flow simplifies to:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y}\right)^2,$$

where $\frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y}\right)^2$ is now considered a constant source term. The initial temperature is T_0 . The temperature at the bottom plate is $T_1 = T_0$. The top plate is adiabatic which means that $\partial T/\partial y=0$.

- 1. Nondimensionalize all the variables and the governing equation (including initial and boundary conditions). Consider a flow with a Reynolds number of 10, take the Prandtl number to be 1, and the Eckert number to be 8.
- 2. Determine the analytical steady state solution. For extra credit: determine the unsteady analytical solution (in terms of a Fourier series) using separation of variables.
- 3. Find the numerical solution using (i) explicit and (ii) implicit first order Euler methods in time and CDS in space. Investigate accuracy (show convergence rates) and stability by considering the effects of Δt , Δy , and $r = \Delta t/(\Delta y)^2$.

Motivate your findings by theoretical characteristics of the error behavior of the methodologies.

To determine errors and convergence rates of the temporal approximation, you may

- use the unsteady analytical solution.
- compares computational solutions with several Δt at at a time t (determined by yourself) in the development of the flow.
- 4. Find the numerical solution using a 4th-order Runge-Kutta method. Study the effect of Δt by running simulations with various Δt .

In the presentation of results, include plots of at least:

- Temperature profile
- Heat flux on the bottom wall versus time
- Global error versus Δt and Δy

Discuss the physics of the solution.

To solve systems of equations use Matlab build-in functions.

You <u>must</u> follow the "Computer Homework" format. Make sure to put your program as an Appendix and not inside your report. Number your figures and refer to the figure number in your report when you discuss the result plotted in the figure.