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A Computational Method for Structurally Nonlinear Joined Wings Based on Modal Derivatives

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Past studies showed that the overconstrained nature of Joined Wings and the strong structural geometric nonlinearities make difficult the use of standard packages of aeroelastic solvers (usually modally reduced and frequency domain based) which have been effectively adopted by the industry for decades. We present here a study on the reduction of the computational cost in presence structural nonlinear effects that cannot be neglected in Joined Wings, even at small angles of attack and attached flow. In particular, a reduced order model is achieved with a basis constituted by vibration modes augmented with the corresponding modal derivatives. The results can be considered excellent when compared to the full order reference solution. However, a convergence test showed that the required number of vectors is relatively high and the basis needs to be often updated to achieve the best performance. More investigations will be necessary for an effective use in the industry and complicate dynamic problems involving the unsteadiness of the aerodynamics.

I. Introduction

THE concept of Joined Wings is not new. In the seventies[1–3] these configurations were proposed for commercial transport and supersonic fighters. The interest grew substantially and several patents in US[4,5] and Europe[6,7] were filed. The interest in the joined-wing concept stemmed from several potential advantages compared to classical cantilevered configurations.[8–12] These advantages are on the stiffness properties, aerodynamic efficiency[13] and superior stability and control characteristics. One of the possible application of Joined Wings is the design of future sustainable configurations such as the PrandtlPlane.[5,8,10,13–18]

Other Joined Wings have been explored in United States such as the Strut-Braced Wings (SBW)[19,20] and Truss Braced Wings (TBW).[21]

On the military side, a possible application of the joined-wing concept is for high altitude surveillance.[22,23]

The potential advantages with respect to cantilevered classical configurations brought the attention of many researchers who investigated several fundamental aspects and problems.[24–29] Several studies showed that structural geometric nonlinearities need to be properly taken into account[30–32] for Joined Wings even for small angles of attack and attached flow. Indeed, one of the main characteristics of typical joined-wing airplanes is the significant forces and moments transferred[33,34] through the joint and the overconstrained nature of the system. This leads to several counter-intuitive behaviors that need to be properly taken into account by the designer. For example, it has been shown in Reference [34] that the lower-to-upper-wing bending stiffness ratio plays a major role in the overall static stability properties and that a stiffer upper wing (the one that is usually compressed during the flight) is not necessarily the best approach to overcome the snap-buckling instability. Additional studies[35] depicted a very complex response: in a regime in which the static response may appear linear, the geometric nonlinearities make the system particularly sensitive to imperfections which determine the possibility of having bi-stable regions for which several equilibrium states

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are possible with a given load. The picture is completed with the potential existence of completely detached (from the main equilibrium branch) set of possible equilibrium states.[36]

The presence of aerodynamic forces which are non-conservative loads in nature, complicates the possible responses even more with other complex phenomena such as the snap-divergence,[37] Limit Cycle Oscillation and probably chaotic dynamic responses.[38]

Ideally, one should have an efficient aeroelastic model tailored for optimization strategies typical of the early conceptual design phases. Thus, reduced order models are particularly important for the preliminary design of these configurations. Unfortunately, in the case of Joined Wings the strong structural nonlinearities makes an efficient reduced order model very difficult to achieve.[39] The procedures developed in References [40], [41], and [42] could be adopted. However, these methods require (for a basis of just a few modes) several *hundreds* of fully nonlinear computationally expensive off-line analyses to evaluate some modal stiffness coefficients adopted to reconstruct the structural behavior. This would not be very practical in the sizing of joined-wing aircrafts and far from an ideal tool for optimization.

An attractive alternative to build effective reduced order models is represented by the adoption of a basis constituted by modes and corresponding *modal derivatives*.[43,44] This study presents an investigation of the performance of this technique on the static nonlinear analysis of Joined Wings is completely assessed. In particular, the following technical points are addressed.

1. Effect of modal derivatives on the quality of the reduced order approximation.
2. Effect of basis updating during the simulation.
3. Effect of the number of vectors that constitute the basis on the effectiveness of the approximation.

II. Reduced Equations of Motion

We consider here the discretized n FE nonlinear dynamic equations of motion of a general tridimensional structure. For the sake of illustrating the potentiality of the proposed method, we do not consider aeroelastic coupling and therefore we assume that the external force term can be expressed as a constant shape scaled by a time dependent function. The governing FE system of equations, together with the boundary conditions, writes:

$$\begin{cases} \mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{g}(\mathbf{u}) = \mathbf{f}\Lambda(t) \\ \\ \mathbf{u}(0) = \mathbf{u}_0 \\ \\ \dot{\mathbf{u}}(0) = \dot{\mathbf{u}}_0 \end{cases} \tag{1}$$

where $\mathbf{u}$ is the generalized displacement vector, $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{g}(\mathbf{u})$ is the nonlinear force vector and $\mathbf{f}$ is the applied load shape multiplied by the time function $\Lambda(t)$. The initial conditions for the displacement and the velocity vector are indicated with $\mathbf{u}_0$ and $\dot{\mathbf{u}}_0$, respectively. We further assume that the nonlinearity of $\mathbf{g}(\mathbf{u})$ is caused by geometrical effects only, i.e. when the displacements are so large that a linear kinematic model does not hold. This is typically the case of thin-walled structures, which can undergo large displacements while staying in the elastic range of the material.

In practical applications, the system of n equations (1) is usually very large. The number of unknowns can be reduced to M, with $m \ll n$, by projecting the displacement field $\mathbf{u}$ on a suitable basis $\mathbf{\Psi}$ of time-independent vectors, as:

$$\mathbf{u} = \mathbf{\Psi}\mathbf{q} \tag{2}$$

where $\mathbf{q}(t)$ is the $M \times 1$ vector of modal amplitudes. The governing equations can then be projected on the chosen basis $\mathbf{\Psi}$ in order to make the residual orthogonal to the subspace in which the solution $\mathbf{q}$ is sought. This results in a reduced system of M non-linear equations:

$$\mathbf{\Psi}^T\mathbf{M}\mathbf{\Psi}\ddot{\mathbf{q}}(t) + \mathbf{\Psi}^T\mathbf{g}(\mathbf{\Psi}\mathbf{q}) = \mathbf{\Psi}^T\mathbf{f}\Lambda(t) \tag{3}$$

or, equivalently,

$$\hat{\mathbf{M}}\ddot{\mathbf{q}}(t) + \hat{\mathbf{g}}(\mathbf{\Psi}\mathbf{q}) = \hat{\mathbf{f}}\Lambda(t) \tag{4}$$

The purpose of this study is to evaluate different reduction strategies for the JW problem. As a first step, we focus on the static problem, i.e. when the inertial forces and damping forces are considered negligible:

$$\hat{\mathbf{g}}(\mathbf{\Psi}\mathbf{q}) = \hat{\mathbf{f}}\Lambda \tag{5}$$

We refer to the numerical solution of the full model as the *full* solution, while the solution of the reduced model will be called *reduced* solution. The key of a good reduction method is to find a suitable basis $\mathbf{\Psi}$ that is able to reproduce the full solution with a good, hopefully controlled, accuracy.

III. Reduction Basis

We discuss is this section how to form the reduction basis $\mathbf{\Psi}$. We propose a basis of vibration modes (VMs) calculated around a given equilibrium position $\mathbf{u}_{eq}$ enriched by the so-called modal derivatives (MDs) and static modes (SMs). These different contributions will be separately discussed.

A. Vibration Modes

Let us consider a static equilibrium position $\mathbf{u}_{eq}$ when the applied load is constant and given by $\mathbf{f}\varphi_{eq}$. We can then linearize the system of equations (1) around such configuration assuming that the motion $\tilde{\mathbf{u}}$ around $\mathbf{u}_{eq}$ is small, i.e. $\mathbf{u} = \mathbf{u}_{eq} + \tilde{\mathbf{u}}$, $\ddot{\mathbf{u}} = \ddot{\tilde{\mathbf{u}}}$. The linearized dynamic equilibrium equations become:

$$\mathbf{M}\ddot{\tilde{\mathbf{u}}} + \mathbf{K}_{eq}\tilde{\mathbf{u}} = \mathbf{f}\tilde{\varphi}(t) \tag{6}$$

where $\tilde{\varphi}(t)$ is a small load variation from φ_{eq}. The tangent stiffness matrix $\mathbf{K}_{eq}$ is defined as:

$$\mathbf{K}_{eq} = \frac{\partial \mathbf{g}}{\partial \mathbf{u}}\bigg|_{\mathbf{u}=\mathbf{u}_{eq}} \tag{7}$$

the eigenvalue problem associated to equation (6) writes:

$$\left(\mathbf{K}_{eq} - \omega_i^2 \mathbf{M}\right)\mathbf{\Phi}_i = \mathbf{0}, \quad i = 1, 2, \ldots, N \tag{8}$$

and its solution provides nVMs $\mathbf{\Phi}_i$ and associated natural frequencies ω_i^2. In linear modal analysis, the displacement vector $\tilde{\mathbf{u}}$ is expressed as a linear combination of VMs as:

$$\tilde{\mathbf{u}} = \sum_{i=1}^{n} \mathbf{\Phi}_i(\mathbf{u}_{eq})q_i \tag{9}$$

We then select a small number m_v of VMs $\mathbf{\Phi}_i$, which can be collected into a matrix $\mathbf{\Phi} = [\mathbf{\Phi}_1, \ldots, \mathbf{\Phi}_{m_v}]$ We discuss the implication of the nonlinearity on the VMs in the next section.

B. Modal Derivatives

The projection of the governing equations on a reduction basis formed by a reduced set of VMs is a well-known technique for linear structural dynamics. The main advantage of this technique is that the resulting reduced model consists of a system of uncoupled equations that can therefore be solved separately. As discussed in the introduction, several attempts has been made to extend the VMs projection for nonlinear analysis. The main limitation of such approach lies in the fact that the VMs basis changes as the configuration of the system changes. It is therefore required to upgrade the basis during the numerical time integration to account for the effect of the nonlinearity. If the basis update is too frequent, the computational advantage in using a MVs reduction basis is lost, since the modes needs to be extracted too often. Moreover, the projection onto a new basi introduces a projection error that accumulates.

For thin-walled structural applications as the one considered in this contribution, the system is usually characterized by significant nonlinear bending-stretching coupling effects that are usually not captured by a reduction basis formed by linear vibration modes only.

When the displacements can not be considered as small, the VMs change with respect to the configuration. We can therefore express the displacement vector $\tilde{\mathbf{u}}$ as

$$\tilde{\mathbf{u}} = \sum_{i=1}^{M} \mathbf{\Phi}_i(\mathbf{u}_{eq} + \tilde{\mathbf{u}}) q_i \tag{10}$$

where the dependence of the VMs on the displacement is highlighted. If equation (10) is expanded in Taylor series around the equilibrium configuration $\mathbf{u}_{eq}$:

$$\tilde{\mathbf{u}} = \sum_{i=1}^{M} \mathbf{\Phi}_i(\mathbf{u}) q_i = \frac{\partial \mathbf{u}}{\partial q_i}\Big|_{\tilde{\mathbf{u}}=\mathbf{0}} q_i + \frac{1}{2} \frac{\partial^2 \mathbf{u}}{\partial q_i \partial q_j}\Big|_{\tilde{\mathbf{u}}=\mathbf{0}} q_i q_j + \dots \tag{11}$$

The derivatives of the displacement vector with respect to the modal amplitudes q_i can be computed from equation (10), and are:

$$\frac{\partial \mathbf{u}}{\partial q_i} = \mathbf{\Phi}_i + \frac{\partial \mathbf{\Phi}_j}{\partial q_i} q_j \tag{12}$$

and

$$\frac{\partial^2 \mathbf{u}}{\partial q_i \partial q_j} = \frac{\partial \mathbf{\Phi}_i}{\partial q_j} + \frac{\partial \mathbf{\Phi}_j}{\partial q_i} + \frac{\partial^2 \mathbf{\Phi}_k}{\partial q_i \partial q_j} q_k \tag{13}$$

.

When evaluated at $\mathbf{u} = \mathbf{u}_{eq}$, they become:

$$\frac{\partial \mathbf{u}}{\partial q_i}\Big|_{\tilde{\mathbf{u}}=\mathbf{0}} = \mathbf{\Phi}_i(\mathbf{u}_{eq}) \tag{14}$$

and

$$\frac{\partial^2 \mathbf{u}}{\partial q_i \partial q_j}\Big|_{\tilde{\mathbf{u}}=\mathbf{0}} = \frac{\partial \mathbf{\Phi}_i}{\partial q_j}(\mathbf{u}_{eq}) + \frac{\partial \mathbf{\Phi}_j}{\partial q_i}(\mathbf{u}_{eq}) \tag{15}$$

The term (14) is the VM, while the term (15) is the MD $\mathbf{\Phi}_{ij}$. It represents how the VM $\mathbf{\Phi}_i$ changes when the system is perturbed in the shape of VM $\mathbf{\Phi}_j$.

A way to compute $\mathbf{\Phi}_{ij}$ is to differentiate the eigenvalue problem (8) with respect to the modal amplitudes.

$$\left[\mathbf{K}_{eq} - \omega_i^2 \mathbf{M}\right] \frac{\partial \mathbf{\Phi}_i}{\partial q_j} + \left[\frac{\partial \mathbf{K}_{eq}}{\partial q_j} - \frac{\partial \omega_i^2}{\partial q_j} \mathbf{M}\right] \mathbf{\Phi}_i = \mathbf{0} \tag{16}$$

It has been shown by[45] and[46] that the terms associated to the mass can be neglected. Numerical experiments have shown that the neglecting of mass related terms does not change the results appreciatively. By doing so, the problem (16) becomes:

$$\mathbf{K}_{eq} \frac{\partial \mathbf{\Phi}_i}{\partial q_j} = -\frac{\partial \mathbf{K}_{eq}}{\partial q_j} \mathbf{\Phi}_i \tag{17}$$

The right-hand side pseudo-force can be calculated at element level an then assembled. It can be shown that the modal derivatives are symmetric, i.e. $\mathbf{\Theta}_{ij} = \mathbf{\Theta}_{ji}$, where $\mathbf{\Theta}_{ij} = \frac{\partial \mathbf{\Phi}_i}{\partial q_j}$. Therefore, given m_v VMs, $m_d = m_v(m_v + 1)/2$ MD can be calculated. Note that the matrix of coefficients can be factorized once for all and only the right-hand sides need to be computed. This can be done at element level and subsequently assembled, see for details.[47] We can collect the MDs into a single matrix $\mathbf{\Theta} = [\mathbf{\Theta}_{11}, \dots, \mathbf{\Theta}_{1m_v}, \dots \mathbf{\Theta}_{ii}, \dots, \mathbf{\Theta}_{im_v}, \dots, \mathbf{\Theta}_{m_v m_v}],\ i = 1, \dots, m_v$.

C. Static modes

The solution of the reduced static nonlinear equilibrium equations inevitably leads to an error unbalance $\mathbf{r}$, defined as:

$$\mathbf{r} = \mathbf{g}(\mathbf{\Phi q}) - \Lambda \mathbf{f}_{ext} \tag{18}$$

where $\mathbf{q}$ is the converged reduced solution for a given load magnitude Λ. If the norm of the residual (normalized with respect to the external forces) exceed a certain treshold, a static mode (SM) can be computed as:

$$\boldsymbol{\Upsilon} = \boldsymbol{K}^{-1}\mathbf{r} \tag{19}$$

where $\mathbf{K}$ is the tangent stiffness matrix already computed ath the last converged load step.

D. Projection Basis

Once a set of m_v VMs $\boldsymbol{\Phi}$ are calculated by solving the eigenvalue problem (8), m_d MDs $\boldsymbol{\Theta}$ can be generated by solving the linear problems (17). A reduction basis including both VMs and MDs can be formed, as:

$$\boldsymbol{\Psi} = [\boldsymbol{\Phi}\ \boldsymbol{\Theta}] \tag{20}$$

when required, the reduction basis can be enriched with a SM also:

$$\boldsymbol{\Psi} = [\boldsymbol{\Phi}\ \boldsymbol{\Theta}\ \boldsymbol{\Upsilon}] \tag{21}$$

The reduction basis Ψ is then orthogonalized, i.e. $\boldsymbol{\Psi}^T\boldsymbol{\Psi} = \mathbf{I}$, to avoid singularity in the reduced stiffness matrix.

IV. Computational Procedures

The solution procedure follows a standard Newton-Raphson algorithm to seek for the modal displacements $\mathbf{q}$. When needed, the reduction basis is recalculated and added to the previous, as indicated in the algorithm 1. After the update, a projection of the obtained solution on the new subspace is needed. As $\boldsymbol{\Psi}$ is orthogonalized, the new modal coordinates are simply obtained by $\mathbf{q} = \boldsymbol{\Psi}^T\mathbf{u}$, where $\mathbf{u}$ is the previously obtained approximation of the full displacement vector. In the following results section, we present in this study a preliminary evaluation of the importance of VMs, MDs, and SMs for the static analysis of the JW.

V. Results

A. Description of the Joined Wing and Full Order Analysis

The Joined Wing investigated in this work has been introduced in Reference [35]. The load, applied to both wings, is represented by a pressure $p = 0.55125\ Kg/(mm \cdot s^2)$, correspondent to the dynamic pressure of air (at sea level) with a speed of 30 m/s. Λ indicates the load fraction: a unitary value means that the entire load is applied to the structure.

The full static response shown in Figure 1b[35] is calculated with a Updated Lagrangian FEM (ULFEM) code developed at San Diego State University and shows the excellent correlation with NASTRAN even in the post-critical regime. The present reduced order model is implemented in a structural solver based on the von Karman moderately large rotations assumption. Correlation with the present Von Karman approach, ULFEM, and NASTRAN is shown in Figure 2 for a load level smaller than the limit point. The Von Karman model is able to accurately reproduce the behavior below the critical point. In this study the post-critical regime is not analyzed, and therefore the von Karman model is adequate.

B. Reduced Order Analysis with no Basis Updating

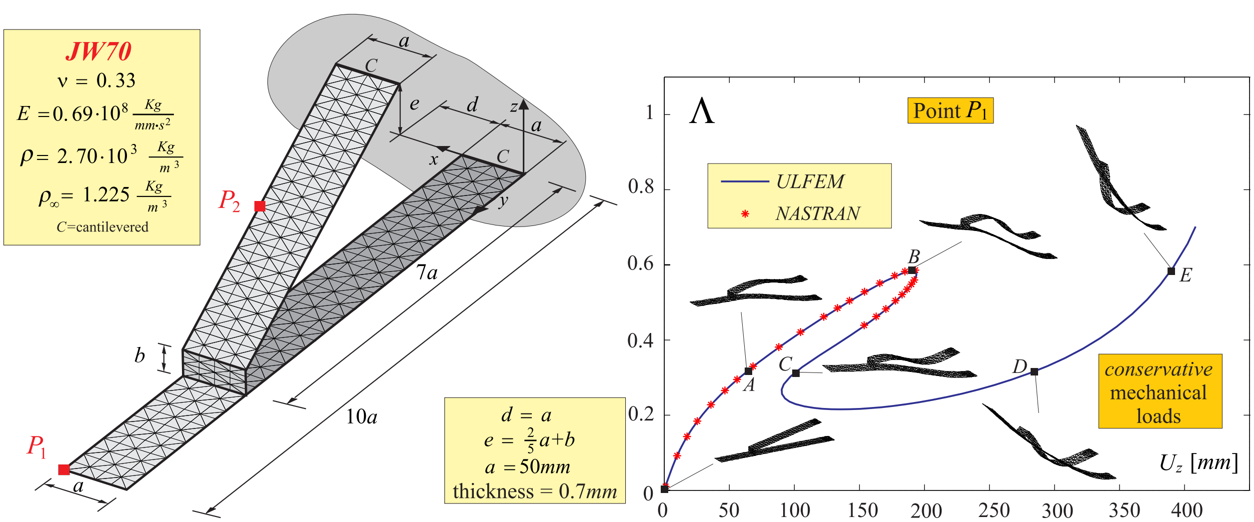
Comparison is made between a basis containing only VMs and a basis containing both VMs and MDs. The performed computational procedures are equal to those described in section IV, however now the basis is *not* updated. This results in the need to use a basis with a relatively large number of vectors to achieve better results. It can be seen in figure 3a that a basis including the first 80% of all VMs hardly approximates the full solution. When MDs are used instead of VMs, the approximated solution converges faster to the full solution with a smaller number of vectors in the basis (Figure 3b). Note that only 1 modal derivative in combination with 1 vibration mode achieves a similar results as 250 VMs in the basis and no MDs. This significant difference may be explained by the fact that the MDs take into account how modes change with respect to imposed displacements (nonlinear effect), whereas the VMs generated in the undeformed configuration only

Algorithm 1 Newton-Raphson algorithm with basis update

Require: initial diplacement $\mathbf{u}_0$,
maximum load factor Λ_{max},
load increment $\Delta\Lambda$,
initial basis $\mathbf{\Psi}$,
residual tolerance $\epsilon_{\mathbf{r}}$,
global residual tolerance $\epsilon_{\mathbf{g}}$

Ensure: equilibrium position $\mathbf{u}(\Lambda)$

form the reduced initial stiffness matrix $\tilde{\mathbf{K}} = \mathbf{\Phi}^T\mathbf{K}(\mathbf{u}_0)\mathbf{\Phi}$
while $\Lambda < \Lambda_{max}$ **do**
 increment load $\Lambda \leftarrow \Lambda + \Delta\Lambda$
 correct the modal displacement $\Delta\mathbf{q} = -\tilde{\mathbf{K}}(\mathbf{q})^{-1}\Delta\Lambda\mathbf{f}_{ext}$
 update the modal displacements $\mathbf{q} \leftarrow \mathbf{q} + \Delta\mathbf{q}$
 evaluate the residual $\tilde{\mathbf{r}}(\mathbf{q}) = \mathbf{\Phi}^T(\mathbf{g}(\mathbf{\Phi}\mathbf{q}) - \Lambda\mathbf{f}_{ext})$
 while $||\tilde{\mathbf{r}}|| > \epsilon_{\mathbf{r}}$ **do**
  evaluate the reduced tangent stiffness matrix $\tilde{\mathbf{K}} = \mathbf{\Phi}^T\mathbf{K}(\mathbf{\Psi}\mathbf{q})\mathbf{\Phi}$
  solve for modal displacement correction $\Delta\mathbf{q} = -\tilde{\mathbf{K}}(\mathbf{q})^{-1}\tilde{\mathbf{r}}$
  update the modal displacements $\mathbf{q} \leftarrow \mathbf{q} + \Delta\mathbf{q}$
  evaluate the residual $\tilde{\mathbf{r}}(\mathbf{q}) = \mathbf{\Phi}^T(\mathbf{g}(\mathbf{\Phi}\mathbf{q}) - \Lambda\mathbf{f}_{ext})$
 end while
 store $\mathbf{u} = \mathbf{\Psi}\mathbf{q}$
 evaluate global residual $\mathbf{r} = \mathbf{g}(\mathbf{\Phi}\mathbf{q}) - \Lambda\mathbf{f}_{ext}$
 if $\dfrac{||\mathbf{r}||}{\Lambda||\mathbf{f}_{ext}||} > \epsilon_{\mathbf{g}}$ **then**
  calculate new vibration modes $\mathbf{\Phi}$
  calculate new modal derivatives $\mathbf{\Theta}$
  calculate new static mode $\mathbf{\Upsilon}$
  update the basis $\Psi \leftarrow [\mathbf{\Psi}\ \mathbf{\Phi}\ \mathbf{\Theta}\ \mathbf{\Upsilon}]$
  orhogonalize $\mathbf{\Psi} = \text{orth}(\mathbf{\Psi})$
  project displacement on new basis $\mathbf{q} = \mathbf{\Psi}^T\mathbf{u}$
  reject last load increment $\Lambda \leftarrow \Lambda - \Delta\Lambda$
 end if
end while



(a) Geometry of the JW70 Joined Wing.

(b) Present Updated Lagrangian Finite Element Formulation (ULFEM) compared with NASTRAN.

Fig. 1: Joined wing used in the present study (see Reference [35]).

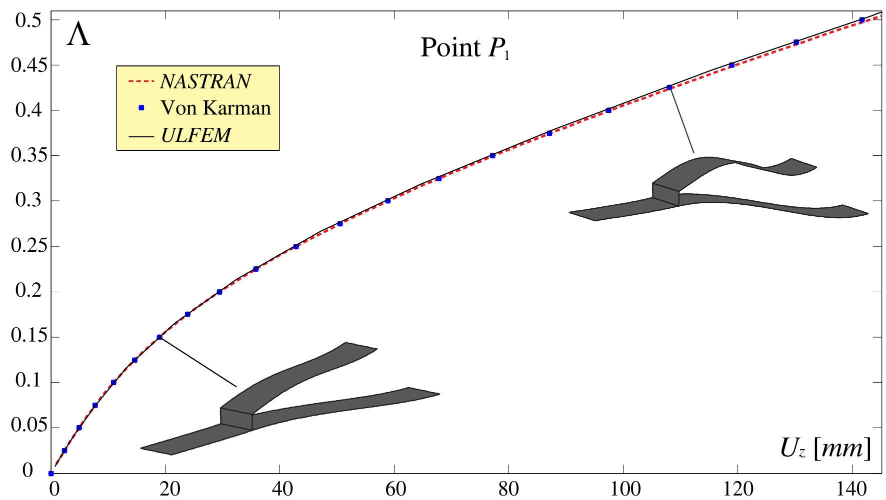
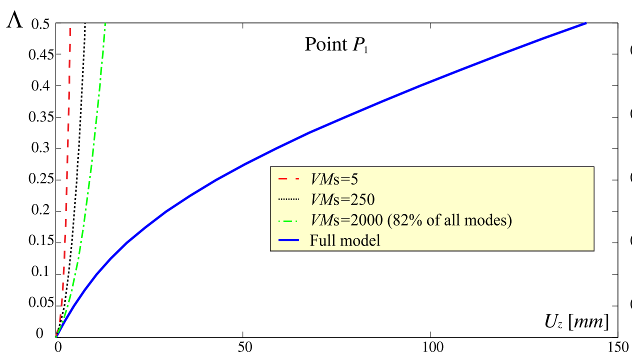
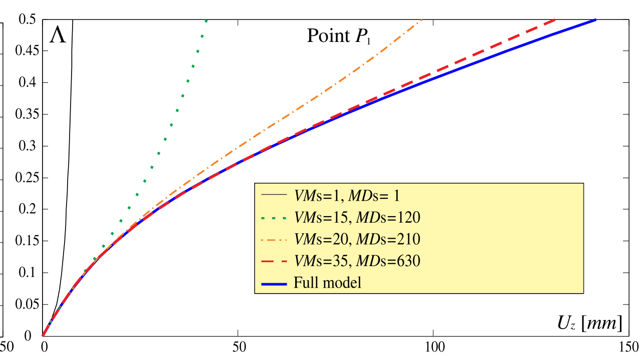


Fig. 2: Full order analysis: comparison between ULFEM, von Karman, and NASTRAN. The approximated von Karman kinematic model reproduces the exact response for load levels below the limit load.



(a) Only VMs used.



(b) Both VMs and MDs used.

Fig. 3: Z-displacement of Point P_1 using different bases. The bases are not updated. The MDs introduce the necessary contribution to describe the geometrically nonlinear effects.

contain linear information. the first two VMs modes and the associated MDs are shown in Figure 4. Note that $\mathbf{\Theta}_{2,1}$ is not shown as this is equal to $\mathbf{\Theta}_{1,2}$ due to symmetry.

C. Reduced Order Analysis with Basis Updating

In this work several variants of updating the basis are explored. The first option is the addition of *static modes* when the basis is updated. The second variation is the rejection of the last step when the basis needs to be updated.

1. VMs VS MDs

The usefulness of MDs in the basis for a static case is evaluated. This is done by a comparison of the simulation results with and without the use of MDs. For a fair evaluation, the number of modes added every basis update is kept equal for both cases. Also the size of the initial basis is kept equal in both cases. The treshold value $\epsilon_{\mathbf{g}}$ is set to a value of 0.025, while the load increment $\Delta\Lambda$ is 0.01. The results are shown in Figure 5a and Figure 5b adding 14 and 20 vectors each basis update respectively. The occurence of a basis update is indicated with a marker. For both cases, the two methods lead to very similar results. For the 14 vectors case, the results are very similar and an equal amount of basis updates (11) are performed, When 20 vectors are added every update, the number of necessary basis updates decreases of about 20%.

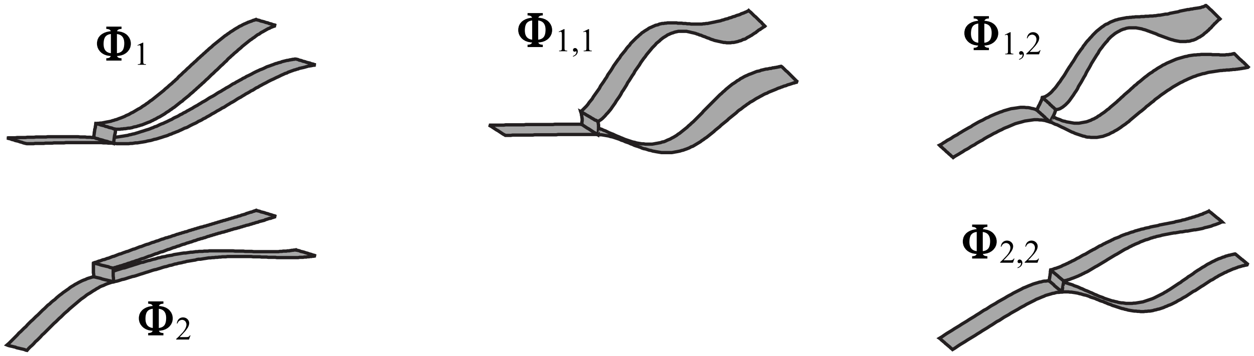
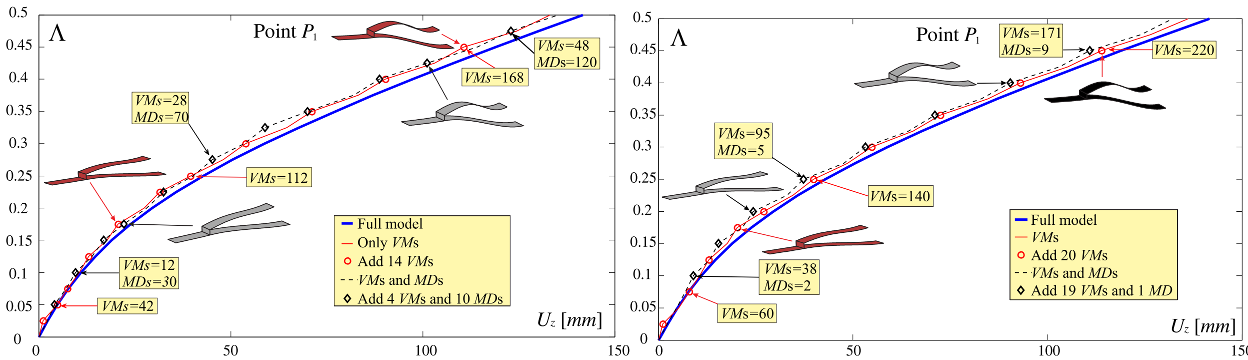


Fig. 4: The first two eigenmodes and associated MDs of the joint wing.



(a) 14 vectors added every basis update.

(b) 20 vectors added every basis update.

Fig. 5: A comparison between the use of only VMs in the basis or also MDs. The occurence of a basis update is indicated with a marker.

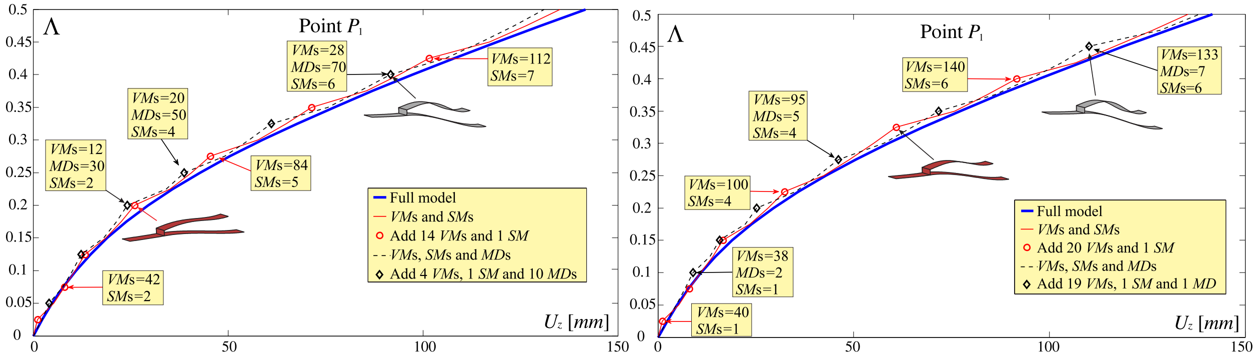
For the 14 vectors case, the results are pretty similar and an equal amount of basis updates (11) are performed. When 20 vectors are added every update, the number of necessary basis updates decreases with 20%. More simulations were performed by the authors, but 20% was the best achieved result. Note that when no MDs are used, the first step already shows a big error because of the lack of non-linear information. Overall, it can be concluded that, for the JW case, the MDs are mostly helpful at low load level, where the strongest nonlinear behavior is observed.

2. Static Modes

Aside from the VMs and MDs, also static modes can be used to enrich the basis. An advantage of static modes is that the computation is very cheap, since the tangential stiffness matrix is already available and factorized. the results relevantly improve. In Figure 6 the effect of the static modes in the basis can be seen. Not only the number of basis update is lowered (up to 40%), but also the solution is more accurately approximated.The addition of SMs is beneficial whether or not MDs are used in the reduction basis.

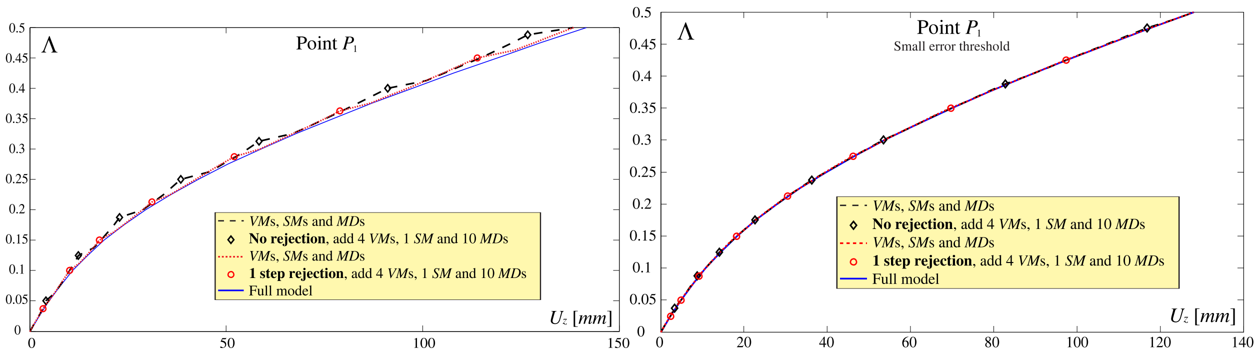
3. Load step rejection

Another way to reduce the error between the full and the reduced model is by rejecting the last loadstep when the basis update occurs. The calculation is then resumed at the last iteration step with the new basis. This effect is clearly seen when the error threshold $\epsilon_{\mathbf{g}}$, forcing a basis to update, isn't too small, see Figure 7a. When the threshold is taken much smaller (0.001) the effect becomes unnoticeable, see Figure 7b. In both cases only the last step is rejected. While clearly beneficial in term of accuracy, the load step rejection lead sometimes to a slight increase in the solution cost, as the step needs to be repeated.



(a) 15 vectors added every basis update. (b) 21 vectors added every basis update.

Fig. 6: Addition of static modes to the basis every time the basis is updated. The number of basis update is substantially reduced.



(a) Default threshold. (b) 2.5 smaller threshold than default.

Fig. 7: Rejecting the last load step when the basis needs to be updated leads to better accuracy of the reduced model.

VI. Conclusions

We evaluated the use of MDs and SMs to enrich a basis formed by VMs to reduce the static analysis of a JW. The MDs prove to be very effective in increasing the accuracy of the reduced basis approximation when calculated around the undeformed configuration, i.e. without basis update. The benefit of MDs seems to greatly reduce when the reduction basis is calculated around a deformed configuration at a relatively high load level. The inclusion of SMs largely improves the solution and leads to a significantly less frequent basis update. Also, the rejection of the last load increment after a basis update yields a better accuracy, but occasionally results in a slightly more costly simulation.

VII. Recommendations and further research

Additional investigation is required for the reduced analysis of the JW. In the future, dynamic and aeroelastic analysis will be considered with the use on reduced models (comparison with the approach presented in Reference [48] will also be pursued). Further investigation needs to be performed for the best strategy for basis update. The number of basis vectors can for instance be kept to a fixed number by discarding non relevant contribution as the simulation proceeds. A load (time) step control can be suggested also, based on the convergence of the reduced solution. Most interesting, the seemingly poor performances of MDs in the neighborhood of a deformed configuration (where the behavior is apparently less nonlinear) need to be further studied. Also, criteria to select an incomplete (and therefore smaller) basis of MDs in line of the simple criterion proposed by[49] can be investigated.

References

[1]Letcher, J., "V-Wings and Diamond-Ring Wings of Minimum Induced Drag," *Journal of Aircraft*, Vol. 8, 1972, pp. 605–607.

[2]Wolkovitch, J., "The Joined Wing Aircraft: an Overview," *Journal of Aircraft*, Vol. 23, 1986.

[3]Lange, R. H., Cahill, J. F., Bradley, E. S., Eudaily, R. R., Jenness, C. M., and Macwilkinson, D. G., "Feasibility Study of the Transonic Biplane Concept for Transport Aircraft Applications," 1974, NASA CR–132462, Lockheed–Georgia Company.

[4]Miranda, L. R., "Boxplane Wing and Aircraft," US Patent 3,834,654, September 1974.

[5]Frediani, A., "Large Dimension Aircraft," US Patent 5,899,409, 1999.

[6]Frediani, A., "Velivolo Biplano ad Ali Contrapposte," Italian Patent FI 2003A000043, 19 February 2003.

[7]Frediani, A., "New Large Aircraft," European Patent EP 0716978B1, 20 March 2002.

[8]Frediani, A., "The Prandtl Wing," *Innovative Configurations and Advanced Concepts for Future Civil Aircraft*, VKI Lecture Series 2005-2006, Von Karman Institute, June 2005, p. [661] p. in various pagings, ISBN: 2-930389-62-1.

[9]Frediani, A., Cipolla, V., and Rizzo, E., "The PrandtlPlane Configuration: Overview on Possible Applications to Civil Aviation," *Variational Analysis and Aerospace Engineering: Mathematical Challenges for Aerospace Design*, edited by G. Buttazzo and A. Frediani, Vol. 66 of *Springer Optimization and Its Applications*, Springer US, 2012, pp. 179–210, 10.1007/978-1-4614-2435-2_8.

[10]Ginneken, D. A. J., Voskuijl, M., Van Tooren, M. J. L., and Frediani, A., "Automated Control Surface Design and Sizing for the Prandtlplane," Presented at the 51st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics & Materials Conference, Orlando, Florida, 12-15 April 2010.

▶[11]Livne, E., "Future of Airplane Aeroelasticity," *Journal of Aircraft*, Vol. 40, 2003, pp. 1066–1092.

▶[12]Livne, E. and Weisshaar, T. A., "Aeroelasticity of Nonconventional Airplane Configurations-Past and Future," *Journal of Aircraft*, Vol. 40, 2003, pp. 1047–1065.

[13]Prandtl, L., "Induced Drag of Multiplanes," Tech. Rep. TN 182, NACA, March 1924.

[14]Chiavacci, L., *Progetto Aerodinamico Preliminare di un Velivolo Ultraleggero tipo PrandtlPlane*, Ms thesis, Università di Pisa, 2006, Advisors: Prof Frediani A., Prof. Casarosa C., PhD Rizzo E.

[15]Frediani, A. and Montanari, G., "Best wing system: an exact solution of the Prandtl's problem," *Variational Analysis and Aerospace Engineering*, Vol. 33 of *Springer Optimization and Its Applications*, Springer New York, 2009, pp. 183–211.

[16]Rizzo, E. and Frediani, A., "Application of Optimisation Algorithms to Aircraft Aerodynamics," *Variational Analysis and Aerospace Engineering*, edited by G. Buttazzo and A. Frediani, Vol. 33 of *Springer Optimization and Its Applications*, Springer New York, 2009, pp. 419–446.

[17]Divoux, N. and Frediani, A., "The Lifting System of a PrandtlPlane, Part 2: Preliminary Study on Flutter Characteristics," *Variational Analysis and Aerospace Engineering: Mathematical Challenges for Aerospace Design*, edited by G. Buttazzo and A. Frediani, Vol. 66 of *Springer Optimization and Its Applications*, Springer US, 2012, pp. 235–267, 10.1007/978-1-4614-2435-2_10.

[18]Frediani, A., Rizzo, E., Bottoni, C., Scanu, J., and Iezzi, G., "A 250 Passenger Prandtlplane Transport Aircraft Preliminary Design," *Aerotecnica Missili e Spazio (AIDAA)*, Vol. 84, 2005.

[19]Sulaeman, E., "Effect of Compressive Force on Aeroelastic Stability of a Strut-Braced Wing," PhD dissertation, November 26, 2001, Blacksburg, Virginia.

▶[20]Gern, F. H., Ko, A., Sulaeman, E., Gundlach, J., Kapania, R. H., and Haftka, R. T., "Multidisciplinary Design Optimization of a Transonic Commerical Transport with Strut-Braced Wing," *Journal of Aircraft*, Vol. 38, 2001, pp. 1006–1014.

▶[21]Gur, O., Bhatia, M., Schetz, J. A., Mason, W. H., Kapania, R. K., and Mavris, D. N., "Design Optimization of a Truss-Braced-Wing Transonic Transport Aircraft," *Journal of Aircraft*, Vol. 47, No. 6, 2010, pp. 1907–1917.

[22]Scott, M., Enke, A., and Flanagan, J., "SensorCraft Free-Flying Aeroservoelastic Model Design and Fabrication," Presented at the 52nd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics & Materials Conference, Denver, Colorado, 4-7 April 2011.

[23]Heeg, J. and Morelli, E., "Evaluation of Simultaneous-Multisine Excitation of the Joined Wing SensorCraft Aeroelastic Wind Tunnel Model," Presented at the 52nd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics & Materials Conference, Denver, Colorado, 4-7 April 2011.

[24]Weisshaar, T. A. and Lee, D. H., "Aeroelastic Tailoring of Joined-Wing Configurations," 43nd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Denver, CO,, 22-25 April 2002.

[25]Reichenbach, E., Castelluccio, M., and B., S., "Joined Wing Sensorcraft Aeroservoelastic Wind Tunnel Test Program," Presented at the 52nd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics & Materials Conference, Denver, Colorado, 4-7 April 2011.

[26]Boston, J., Swenson, E., and Kunz, D., "Experimens with Geometric Non-Linear Coupling for Analytical Validation," Presented at the 51st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics & Materials Conference, Orlando, Florida, 12-15 April 2010.

▶[27]Paletta, N., Belardo, M., and Pecora, M., "Load Alleviation on a Joined-Wing Unmanned Aicraft," *Journal of Aircraft*, Vol. 47, 2010, pp. 2005–2016.

▶[28]Sotoudeh, Z. and Hodges, D. H., "Incremental Method for Structural Analysis of Joined-Wing Aircraft," *Journal of Aircraft*, Vol. 48, 2011, pp. 1588–1601.

[29]Su, W., *Coupled Nonlinear Aeroelasticity and Flight Dynamics of fully Flexible Aircraft*, Phd thesis, University of Michigan, 2008.

[30]Blair, M., Canfield, R. A., and Roberts Jr., R. W., "Joined-Wing Aeroelastic Design with Geometric Nonlinearity," *Journal of Aircraft*, Vol. 42, 2005.

[31]Kim, Y. I., Park, G. J., Kolonay, R. M., Blair, M., and Canfield, R. A., "Nonlinear Response Structural Optimization of a Joined Wing Using Equivalent Loads," *AIAA Journal*, Vol. 46, 2008.

[32]Chambers, J. R., *Innovation in Flight: Research of the NASA Langley Research Center on Revolutionary Advanced Concepts for Aeronautics*, No. 39 in Monograph in Aerospace History, NASA, November 2005, NASA SP 2005-4539.

[33]Stearman, R., Lin, H.-H., and Jhou, J., "Influence of Joint Fixity on the Aeroelastic Characteristics of a Joined Wing Structure," No. AIAA-90-0980-CP, AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, 31st, Long Beach, CA, American Institute of Aeronautics and Astronautics, Washington, DC, April 2-4 1990, pp. 1442–1454.

[34]Cavallaro, R., Demasi, L., and Passariello, A., "Nonlinear Analysis of PrandtlPlane Joined Wings: Effects of Anisotropy," 2012, Manuscript submitted for publication.

[35]Cavallaro, R., Demasi, L., and Bertuccelli, F., "Risks of Linear Design of Joined Wings: a Nonlinear Dynamic Perspective in the Presence of Follower Forces," No. AIAA 2013-1558, 54rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Boston, Massachusetts, American Institute of Aeronautics and Astronautics, 8-11 April 2013.

[36]Cavallaro, R., Demasi, L., Bertuccelli, F., and Benson, D. J., "Risks of Linear Design of Joined Wings: a Nonlinear Dynamic Perspective in the Presence of Follower Forces," *CEAS Aeronautical Journal*, 2013, Submitted for Pubblication.

[37]Demasi, L., Cavallaro, R., and Bertuccelli, F., "Post-Critical Analysis of Joined Wings: the Concept of Snap-Divergence as a Characterization of the Instability," No. AIAA 2013-1559, 54rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Boston, Massachusetts, American Institute of Aeronautics and Astronautics, 8-11 April 2013.

[38]Cavallaro, R., Iannelli, A., Demasi, L., and Razon, A. M., "Phenomenology of Nonlinear Aeroelastic Responses of highly Deformable Joined-wings Configurations," 55th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, National Harbor, Maryland, American Institute of Aeronautics and Astronautics, 13-17 January 2014.

[39]Demasi, L. and Palacios, A., "A Reduced Order Nonlinear Aeroelastic Analysis of Joined Wings Based on the Proper Orthogonal Decomposition," Presented at the 51st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics & Materials Conference, Orlando, Florida, 12-15 April 2010.

[40]Muravyov, A. A. and Rizzi, S. A., "Determination of Nonlinear Stiffness with Application to Random Vibration of Geometrically Nonlinear Structures," *Computers and Structures*, Vol. 81, 2003, pp. 1513–1523.

[41]Rizzi, S. A. and Przekop, A., "Estimation of Sonic Fatigue by Reduced-Order Finite Element Based Analyses," Presented at the IX International Conference on Recent Advances in Structural Dynamics, Southampton, UK, 17-19 July 2006.

[42]Liu, D. D., Chen, P. C., Zhang, Z., Wang, Z., Yang, S., Lee, D. H., Mignolet, M., Kim, K., Liu, F., Beran, P., and lindsley, N., "Continuous Dynamic Simulation for Morphing Wing Aeroelasticity," Presented at the 50th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics & Materials Conference, Palm Springs, California, 4-7 May 2009.

[43]Tiso, P. and Jansen, E., "A Finite Element Based Reduction Method for Nonlinear Dynamics of Structures," Presented at the 46th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics & Materials Conference, Austin, Texas, 18-21 April 2005.

[44]Idelsohn, S. R. and Cardona, A., "A reduction method for nonlinear structural dynamic analysis," *Computer methods in applied mechanics and engineering*, Vol. 49, 1985, pp. 253–279.

[45]Idelsohn, S. R. and Cardona, A., "A reduction method for nonlinear structural dynamic analysis," *Computer Methods in Applied Mechanics and Engineering*, Vol. 49, 1985, pp. 253–279.

[46]Slaats, P. M. A., de Jong, J., and Sauren, A. A. H. J., "Model reduction tools for nonlinear structural dynamics," *Computer & Structures*, Vol. 54, 1995, pp. 1155–1171.

[47]Tiso, P., Jansen, E., and Abdalla, M., "Reduction Method for Finite Element Nonlinear Dynamic Analysis of Shells," *AIAA Journal*, Vol. 49, No. 10, 2012/11/01 2011, pp. 2295–2304.

[48]Phlipot, G., Wang, X. Q., Mignolet, M. P., Demasi, L., and Cavallaro, R., "Reduced Order Modeling for the Nonlinear Geometric Response of Some Joined Wings," AIAA Science and Technology Forum and Exposition 2014, National Harbor, MD, 13-17 January 2014.

[49]Tiso, P. and Rixen, D. J., "Optimal second order reduction basis selection for nonlinear transient analysis," *29th IMAC, Jacksonville, FL USA*, 2011.