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Coding Efficiency Evaluation of RVLC Codes

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Abstract— **Variable length coding (VLC) has become an important part of visual compression standards such as JPEG, MPEG, and H.26x. Recently, reversible variable length coding is being used to improve the robustness against bit errors. The coding efficiency performance of different reversible VLC (RVLC) schemes depends on several factors such as source distribution, quantizer step, and codeword length distribution. In this paper, we develop a mathematical approach to calculate the efficiency of RVLC code table for coding symbols with generalized Gaussian distribution.**

I. INTRODUCTION

Video signals transmitted over error-prone wireless channels are often corrupted by bit errors. Because the compressed data is very sensitive to errors, error resiliency schemes are generally used to achieve graceful degradation. The reversible variable length code (RVLC) has been used to improve error resiliency in the MPEG-4 [1], and H.263 [2] video coding standards. As the RVLC coded data can be decoded in the forward as well as backward directions, better data recovery is achieved in the presence of bit errors, as compared to classical VLCs (e.g., Huffman codes). RVLC schemes have been developed by several groups of researchers, such as Toshiba (which has been adopted by MPEG-4) [3], Tsai and Wu [4], and Villasenor et al. [5, 8]. The coding efficiency of RVLC codes varies for different types of sources. However, no generalized mathematical framework has been reported so far to compare the coding efficiency performance of the RVLC techniques. In this paper, we develop a mathematical framework to study the coding efficiency of RVLC schemes for the generalized Gaussian (GG) source and MPEG-4 video sequences.

We organize the rest of the paper as follows. The proposed coding efficiency evaluation approach is presented in Section II. Section III presents the coding efficiency evaluation of five RVLC schemes (Golomb-Rice (GR) [8], exponential-Golomb (EG) [8], Toshiba (MPEG-4) [3], Tsai and Wu (T&W) [4], and Lakovic and Villasenor (L&V) [5] RVLC), followed by the conclusions in Section IV.

II. PROPOSED EVALUATION APPROACH

We make the following assumptions to develop the coding efficiency evaluation approach:

i) A uniform quantizer with step size δ, and zero dead zone is used before applying the VLC.
ii) x is the value of a source symbol, which is to be encoded.
iii) X is a real number space with $0 \le X < \infty$, and the source symbol lying in the interval $[X_k, X_{k+1})$ or $[k\delta, (k+1)\delta]$ produces the symbol-k.

A. Coding Efficiency Function for General Sources

Let the probability of the source symbol (x) to be in the interval $[X_k, X_{k+1})$ be denoted by P_k, i.e., $P_k = \Pr(X_k \le x < X_{k+1}) = \int_{k\delta}^{(k+1)\delta} p(x)dx$. The entropy h (in bits/symbol) of the source is then given by $h = -\sum_k P_k \log_2 P_k$. Let l_k represent the length of the codeword (in bits) for symbol-k. The average codeword length is then given by $M = \sum_k P_k \cdot l_k = P^T \cdot L$ where, $P^T = [P_0 \quad P_1 \ldots \qquad \ldots]$ and $L = [l_0 \quad l_1 \ldots \qquad \ldots]^T$. For a source quantized by a quantization step δ, we can calculate the coding efficiency for a codeword length table L as follows.

$$f(\delta, L) = \frac{h}{M} = \frac{h}{P^T \cdot L} = \frac{-\sum_{k=0}^{\infty} P_k \log_2 P_k}{\sum_{k=0}^{\infty} P_k l_k} \qquad (1)$$

Eq. (1) calculates the coding efficiency of RVLC for a source with arbitrary distribution. However, the transform coefficients such as DCT and wavelets are known to have generalized Gaussian distribution [6]. Therefore, in the next subsection, we develop a mathematical approach to calculate the coding efficiency of an RVLC scheme for a generalized Gaussian (GG) source.

B. Coding Efficiency Function for GG Source

The double-sided GG pdf can be written as [6]:

$$p(x) = C_1 \exp(-C_2 |x|^v) \text{ where } C_1 = \frac{v\eta(\sigma, v)}{2\Gamma(1/v)},$$

$$C_2 = [\eta(\sigma, v)]^v \text{ and } \eta(\sigma, v) = \frac{1}{\sigma}\left[\frac{\Gamma(3/v)}{\Gamma(1/v)}\right]^{\frac{1}{2}}. \text{ and}$$

$\Gamma(\cdot)$ is the *gamma* function. The non-negative GG (NNGG) pdf can be expressed as:

$$p(x) = 2C_1 \exp(-C_2 x^v), x \ge 0 \qquad (2).$$

The GG function can be parameterized by shape parameter (ν) and standard deviation (σ). The NNGG pdfs for different ν are shown in Fig. 1. It is observed that a small shape parameter (ν) represents a *pdf* with sharper peak. As ν increases (with a constant σ), the pdf becomes flat (*e.g.*, ν ≥ 0.7) and looks closer to the uniform density function. Using method discussed in [7], we show in Fig. 2 that the NNGG pdf (with ν = 0.575, σ =1) can closely match the distribution of DCT coefficients in MPEG-4 'akiyo' video sequence. Here we have used absolute amplitude of DCT coefficients as the H.263 and MPEG-4 video coding standards represent the sign of an RVLC codeword in a separate 'sign' bit. Other sources can be similarly shown to match the GG pdf for particular values of ν and σ.

From eq. (2), we can represent an NNGG pdf (parameterized by υ and σ) as,

$$p_{v,\sigma}(x) = \frac{v}{\Gamma(1/v)}\alpha(v)\cdot\frac{1}{\sigma}\cdot\exp\left[-\left(\frac{x}{\sigma}\alpha(v)\right)^{v}\right] \quad (3),$$

where $\alpha(v) = \left[\frac{\Gamma(3/v)}{\Gamma(1/v)}\right]^{\frac{1}{2}}$. From eq. (3), $p_{v,\sigma}(x)$ can also be written as,

$$p_{v,\sigma}(x) = \frac{v}{\Gamma(1/v)}\alpha(v)\cdot\frac{c}{c\sigma}\cdot\exp\left[-\left(\frac{cx}{c\sigma}\alpha(v)\right)^{v}\right] = cp_{v,c\sigma}(cx) \quad (4),$$

where c is a positive constant. From eq. (4), we can make the following proposition.

Proposition 1: If $h_k(\delta,\sigma)$ is the entropy of an NNGG source in interval [kδ, (k+1)δ), quantized by step δ and parameterized by υ and σ, then $h_k(\delta,\sigma) = h_k(c\delta,c\sigma)$, for a fixed value of υ.

Proof:

$$\begin{aligned} h_k(\delta,\sigma) &= -\Pr(X_k < x < X_{k+1})\cdot\log_2\Pr(X_k < x < X_{k+1}) \\ &= -\int_{k\delta}^{(k+1)\delta} p_{v,\sigma}(x)dx\cdot\log_2\left(\int_{k\delta}^{(k+1)\delta} p_{v,\sigma}(x)dx\right) \\ &= -\int_{k\delta}^{(k+1)\delta} cp_{v,c\sigma}(cx)dx\cdot\log_2\left(\int_{k\delta}^{(k+1)\delta} cp_{v,c\sigma}(cx)dx\right) \\ &= -\int_{kc\delta}^{(k+1)c\delta} p_{v,c\sigma}(x)dx\cdot\log_2\left(\int_{kc\delta}^{(k+1)c\delta} p_{v,c\sigma}(x)dx\right) \\ &= h_k(c\delta,c\sigma) \end{aligned}$$

Q.E.D

The physical significance of the Proposition 1 is shown in Fig. 3, where two NNGG *pdf*'s (for υ=0.5, σ=1, and υ=0.5, σ=2) are shown. Note that $\text{area}_A = \text{area}_B + \text{area}_C$. Therefore, $\Pr_{\sigma=1}(0.04 \le x < 0.06) = \Pr_{\sigma=2}(0.08 \le x < 0.12)$ and thus $h_2(0.02,1) = h_2(0.04,2)$. Since we have $h_k(\delta,\sigma) = h_k(c\delta,c\sigma)$, $h_k(\delta,\sigma)$ can be represented by $h_k(\delta/\sigma)$.

One the other hand, using eq. (3), we can express the probability of an NNGG source (parameterized by υ and σ) in the interval $X_k \le x < X_{k+1}$ as,

$$\Pr_k(\delta,\sigma) = \frac{v}{\Gamma(1/v)}\alpha(v)\int_{k\delta}^{(k+1)\delta}\frac{1}{\sigma}\cdot\exp\left[-\left(\frac{x}{\sigma}\alpha(v)\right)^{v}\right]dx \quad (5).$$

In order to cancel the effect of standard deviation (σ) in eq. (5), we can use the normalized quantizer step (δ/σ) by substituting x/σ for x in the integral as,

$$\Pr_k(\frac{\delta}{\sigma}) = \frac{v\alpha(v)}{\Gamma(1/v)}\int_{k\delta/\sigma}^{(k+1)\delta/\sigma}\exp[-(x\alpha(v))^{v}]\ dx \quad (6).$$

From eq. (1) and (6), the coding efficiency function of an RVLC scheme (with certain codeword length table given by L) for a GG source (parameterized by υ and the normalized quantizer step δ/σ) can be expressed as,

$$f(v,\delta/\sigma,L) = \frac{-\sum_{k=0}^{\infty} h_k(\frac{\delta}{\sigma})}{\sum_{k=0}^{\infty}\left[\Pr_k(\frac{\delta}{\sigma})\cdot l_k\right]} \quad (7).$$

If the shape parameter (ν) and the codeword length table (L) are given, the coding efficiency function in eq. (7) can be simplified to f (δ/σ). Eq. (7) is thus derived from eq. (1) in order to consider the quantization of source and GG function parameters (ν, σ), which are useful in most lossy compression schemes.

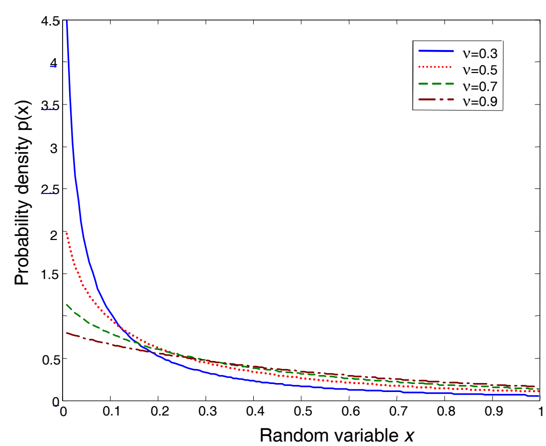

Fig. 1: The shapes of NNGG pdf with σ = 1.

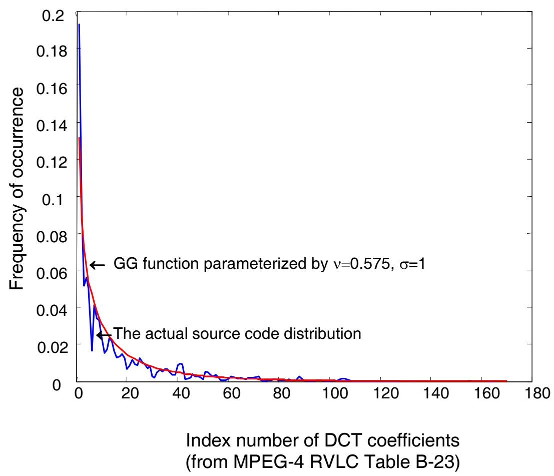

Fig. 2: NNGG pdf matched to the source distribution of codewords (i.e., quantized DCT coefficients) in MPEG-4 'akiyo' video sequence.

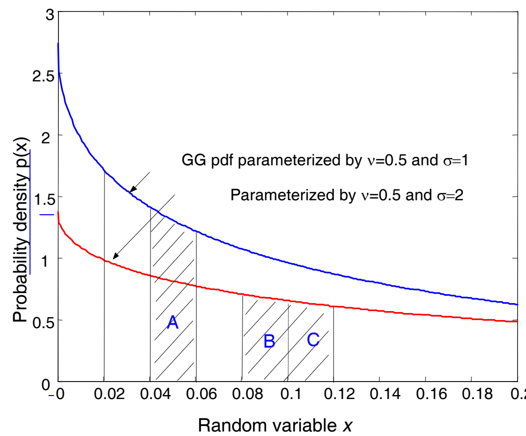

Fig. 3: Two NNGG pdfs parameterized by $\nu = 0.5, \sigma = 1$ and $\nu = 0.5, \sigma = 2$, respectively; $area_A = area_B + area_C$.

C. *Coding Efficiency Function for GG Source*

To minimize expected codeword length, the length distribution of RVLC codewords should be according to the corresponding source symbol entropy. As shown in Section II.B, the entropy relates to the source distribution and quantizer step (δ). The codeword table is constructed for a given source, by assigning codeword with appropriate length to each source symbol, in order to maximize the coding efficiency in eq. (1). Since NNGG is a monotonically decreasing function, we have $P_0>P_1>P_2$……. Accordingly, we let $l_0 \le l_1 \le l_2$......to minimize the overall expectation of codeword lengths. Some RVLC schemes, such as Golomb-Rice [8] and Toshiba RVLC [3] have fixed prefix length distribution that makes them suitable for certain source distributions. For example, Golomb-Rice RVLC is suited to exponential sources [8]. Similarly, Toshiba RVLC scheme has been used to encode DCT coefficients in MPEG-4 [3]. Other RVLC schemes proposed by Tsai and Wu (T&W) [4] and Lakovic and Villasenor (L&V) [5] use Huffman code as reference to match the source distribution. They can therefore adapt themselves to many types of sources. The generalized Gaussian sources [6] parameterized by ν (0.3, 0.5 and 0.7) and σ (1) have been used to generate the T&W and L&V RVLC tables. Moreover, an RVLC scheme can be parameterized by choosing a suitable suffix length (K) to match the source distributions.

III. Performance Evaluation

The coding efficiency evaluation approach for RVLC schemes was developed in Section II. In this section, we present examples to study the coding efficiency of five RVLC scheme(s) for GG source and MPEG-4 sequences.

In Fig. 4, we use eq. (7) to evaluate the coding efficiency of a few RVLC schemes for an NNGG source (with shape parameter ν = 0.5), by changing the normalized quantizer step δ/σ. The coding efficiency of T&W (ν=0.3, K=1) scheme is very close to that of MPEG-4 RVLC scheme for the normalized quantizer step $0.01 < \delta/\sigma < 1$. For $0.025 < \delta/\sigma < 0.063$, the MPEG-4 RVLC performs better than different T&W RVLCs, whereas some of the T&W RVLCs have better performance for other values of δ/σ. In Fig. 5, we show the coding efficiency of the Toshiba MPEG-4 RVLC scheme for coding NNGG sources (parameterized by ν). The RVLC scheme has more than 95% coding efficiency for ν = 0.5 and 0.7, when δ/σ is between 0.032 and 0.1. The variation in coding efficiency with δ/σ is due to variation of the code length compared with the entropy. From the above, we see that the coding efficiency of an RVLC scheme depends on δ/σ and source distribution. It is therefore possible to *improve* the coding efficiency of an RVLC scheme, for a given GG source distribution, by tuning its parameters such as K (suffix length), ν, and σ.

We also used the five RVLCs with different parameters (ν, K) for coding MPEG-4 video test sequences 'akiyo' and 'foreman' (352x288, 100 frames at 10 fps and fixed quantizer size of 6). For this, we substituted RVLC codewords in MPEG-4 Table B-23 [1] by using EG, GR, T&W, and L&V RVLCs. The same combinations of Last, Run, and Level were used as in the Table B-23. Table I shows the relative coding efficiency (ratio of the compressed file size obtained by using another RVLC scheme to the MPEG-4 RVLC scheme in percentage) of various RVLCs for two MPEG-4 video sequences, 'akiyo' and 'foreman'. Here the Toshiba RVLC adopted in MPEG-4 has the best coding efficiency. The performance of the T&W RVLC (ν=0.3 and K=1) is very close to that of the MPEG-4 RVLC. The T&W as well as L&W RVLC with ν=0.3 and K=1 has better coding efficiency than with ν=0.5 and K=0. The EG and GR RVLC schemes show the best coding efficiency in their class, with K=3. Their coding efficiency is, however, lower than that of T&W and L&V RVLCs. These schemes can be arranged in the decreasing order of their coding efficiency as: MPEG-4, T&W (ν=0.3, K=1), L&V (ν=0.3, K=1), EG (K=3) and GR (K=3). Please note that this order has one combination of parameters(s) for each of the five RVLC schemes.

IV. Conclusion

We introduced a generalized approach to evaluate the coding efficiency of an RVLC scheme. This approach is also applicable to any variable length code (VLC). It has been shown analytically that the coding efficiency of an RVLC scheme is related to the source distribution, quantizer step, and codeword length distribution. An RVLC scheme can be parameterized by the suffix length (K), shape parameter v, and σ to match the source distributions. This has also been verified by the experimental results.

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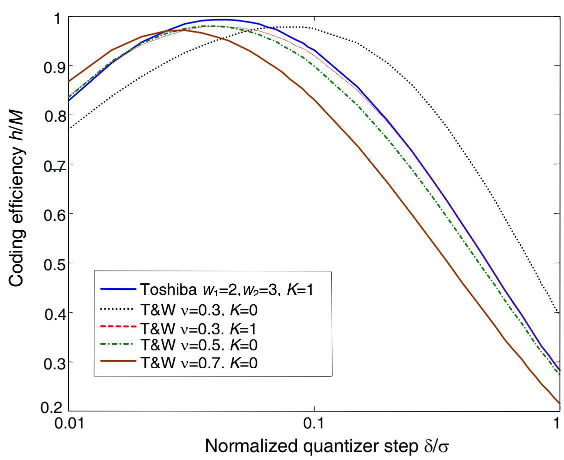

Fig. 4: Efficiency of the RVLCs for an NNGG source with v=0.5.

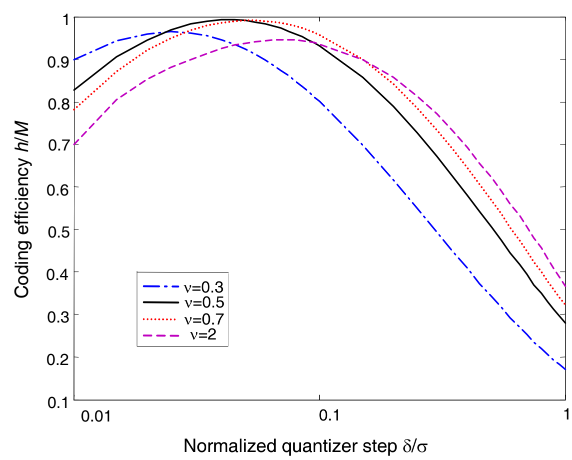

Fig. 5: Efficiency of MPEG-4 RVLC scheme for different NNGG pdfs.

Table I: Coding efficiency of RVLCs for MPEG-4 video sequence, akiyo and foreman (in brackets).

RVLC	***v***	***K***	**Coding Efficiency**	**RVLC**	***v***	***K***	**Coding Efficiency**
			Akiyo (Foreman)				**Akiyo (Foreman)**
MPEG-4	-	1	100.00% (100.00%)	L&V	0.7	0	104.43% (103.65%)
T&W	0.3	1	100.96% (100.19%)	GR	-	3	104.83% (104.79%)
T&W	0.5	0	101.41% (100.61%)	GR	-	4	105.78% (104.82%)
L&V	0.3	1	102.60% (101.60%)	L&V	0.3	0	105.82% (104.64%)
T&W	0.7	0	102.85% (101.60%)	EG	-	1	106.43% (106.38%)
L&V	0.5	0	102.92% (102.04%)	T&W	0.7	1	107.88% (106.50%)
T&W	0.3	2	102.97% (101.97%)	EG	-	4	107.97% (106.66%)
T&W	0.5	1	103.25% (102.30%)	L&V	0.7	1	109.45% (107.70%)
EG	-	3	103.37% (102.14%)	T&W	0.5	2	110.13% (108.49%)
L&V	0.3	2	103.52% (102.57%)	L&V	0.5	2	110.24% (108.69%)
EG	-	2	103.66% (103.70%)	EG	-	0	113.65% (112.92%)
L&V	0.5	1	103.78% (102.98%)	GR	-	2	119.87% (121.41%)
T&W	0.3	0	104.06% (102.98%)	GR	-	5	130.41% (128.28%)